The Parallel Implementation of the Adaptive Mesh Technology in Poroelasticity Problems

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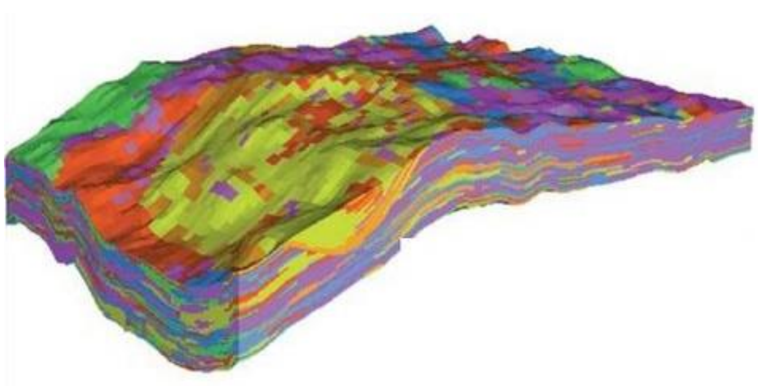
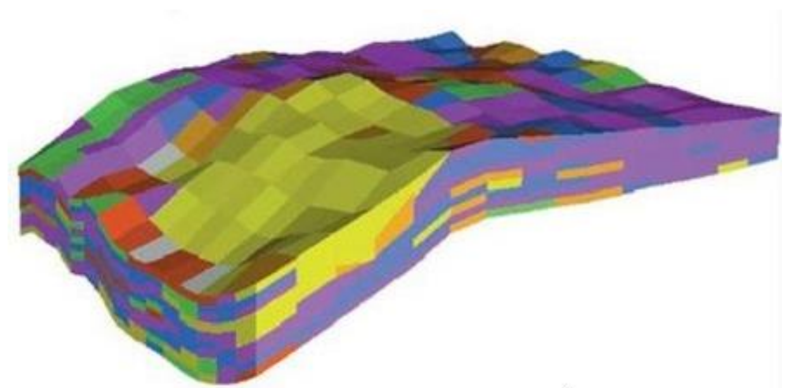
Modeling of Oil&Gas Reservoirs

- The frontier of modeling techniques: proxy models – reduced amount of cells using the so-called upscaling of the cells.
- Building separate models for hydrodynamics and geomechanics increases the risks to get negative impact in the development process => necessary to solve the problems coupled
- There is no universal tool to solve poroelastic problems of sufficiently a large size
- It is necessary to choose such an approach that the computational grid would have the smallest size (by the number of grid nodes), and the accuracy of calculations would be maximum possible





Some Aspects of Adaptive Grids

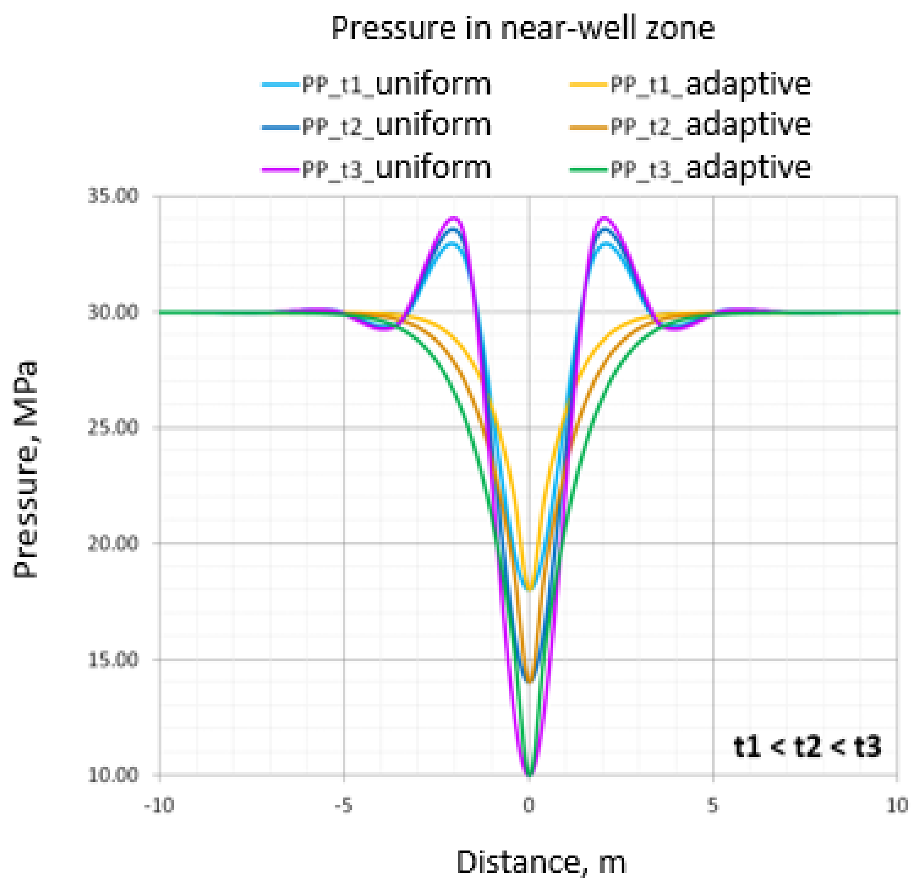
- **R-adaptation technique:** the reduction in the size of some elements occurs due to an increase in the size of others
- Total number of elements remain **constant**
- **Uniform loading** of computational nodes in the course of parallel computing
- The commercial software tries to solve a problem of an accuracy by increasing the number of cells in the zones of a rapid change of a certain field => significant increase in calculation time

The idea of this research is to present a technique that is appropriate for solving large poroelastic problems with the help of a supercomputer with an effective strategy for adapting computational grid for parallelization.

Problem of Fluid Inflow into a Well

$$\int_V \frac{\varphi\,\rho_f}{K}\frac{\partial p}{\partial t}\,\delta p\,dV + \int_V \rho_f \frac{k}{\mu}\nabla p \cdot \nabla \delta p\,dV = \int_{A^m} m \delta p\,dA,$$

where φ – porosity, ρ_f – fluid density, K – compressibility of a reservoir fluid, k – reservoir permeability, μ – viscosity of the reservoir fluid, p – desired pressure, m – mass flow through the part A^m of the outer boundary A



Permeability:

K=5e-16 $м^2$ (0.5 mD)

Porosity:

0.15

Dynamic fluid viscosity:

2.837e-5 Pa*s

Density:

850 kg/m^3

Fluid compression bulk module:

2e9 Pa

Initial conditions:

3e7 Pa

Boundary conditions (∞):

3e7 Pa

Number of elements:

N=112

Algorithm of Adaptive Mesh Construction

- Two main methods for constructing an adaptive mesh:
  - **Principle of an equidistributing grid** (error in estimating a desired function is the same on each element)
  - **Finding a solution of the grid differential equation**
- Both methods lead to a system of coupled equations for determining:
  - position of a moving grid
  - distribution function of a certain physical quantity
- In practice, a variational method are used. The **Euler-Lagrange equations** are written down with a “grid” functional of a special form.

Algorithm of Adaptive Mesh Construction

In the case of a variation approach, the general form of the Euler-Lagrange equation can be obtained in the following form:

$$-\nabla \cdot \left[\frac{\partial F}{\partial \boldsymbol{a}^i} - J\frac{\partial F}{\partial J}\boldsymbol{a}_i\right] = 0, \qquad i = 1,2,3$$

where the corresponding functional is as follows:

$$I[\xi] = \int_{\Omega} F(\boldsymbol{a}^1, \boldsymbol{a}^2, \boldsymbol{a}^3, J, \boldsymbol{x})\, dx.$$

It is assumed that J is the Jacobian of the transformation, and the corresponding vectors $\boldsymbol{a}^i$ are the columns of the inverse Jacobi matrix:

$$\boldsymbol{J} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{\xi}} = \frac{\partial(x_1, x_2, x_3)}{\partial(\xi_1, \xi_2, \xi_3)} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3].$$

For practical purposes, the function F in the integral can be represented as follows:

$$F(\boldsymbol{a}^1, \boldsymbol{a}^2, \boldsymbol{a}^3, J, \boldsymbol{x}) = F_1(\rho, \beta) + F_2(\rho, J)$$

$$F_1(\rho, \beta) = \frac{1}{2}\sum\nolimits_i (\nabla\xi_i)^T M^{-1} \nabla\xi_i = \frac{1}{2}\beta,\ \ F_2(\rho, J) = 0$$

$$M = w(\boldsymbol{x})I,\ \ w = \sqrt{1 + |\nabla p|^2}$$

where $w(\boldsymbol{x})$ is the function that determines the «density» of grid lines used to solve the physical equation.

Algorithm of Adaptive Mesh Construction

After performing rather a cumbersome chain of transformations aimed at changing the roles of the independent and dependent variables, we can obtain a compact form of the «grid» differential equation for determining the function $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{\xi})$:

$$\sum_{i,j} A_{i,j} \frac{\partial^2 \boldsymbol{x}}{\partial \xi_i \partial \xi_j} + \sum_i B_i \frac{\partial \boldsymbol{x}}{\partial \xi_i} = 0.$$

In this equation:

$$A_{ij} = \left(\left(\boldsymbol{a}^i\right)^T M^{-1} \boldsymbol{a}^j \right) I$$

$$B_i = I \sum_k \left(\left(\boldsymbol{a}^k\right)^T \frac{\partial M^{-1}}{\partial \xi_k} \boldsymbol{a}^i \right).$$

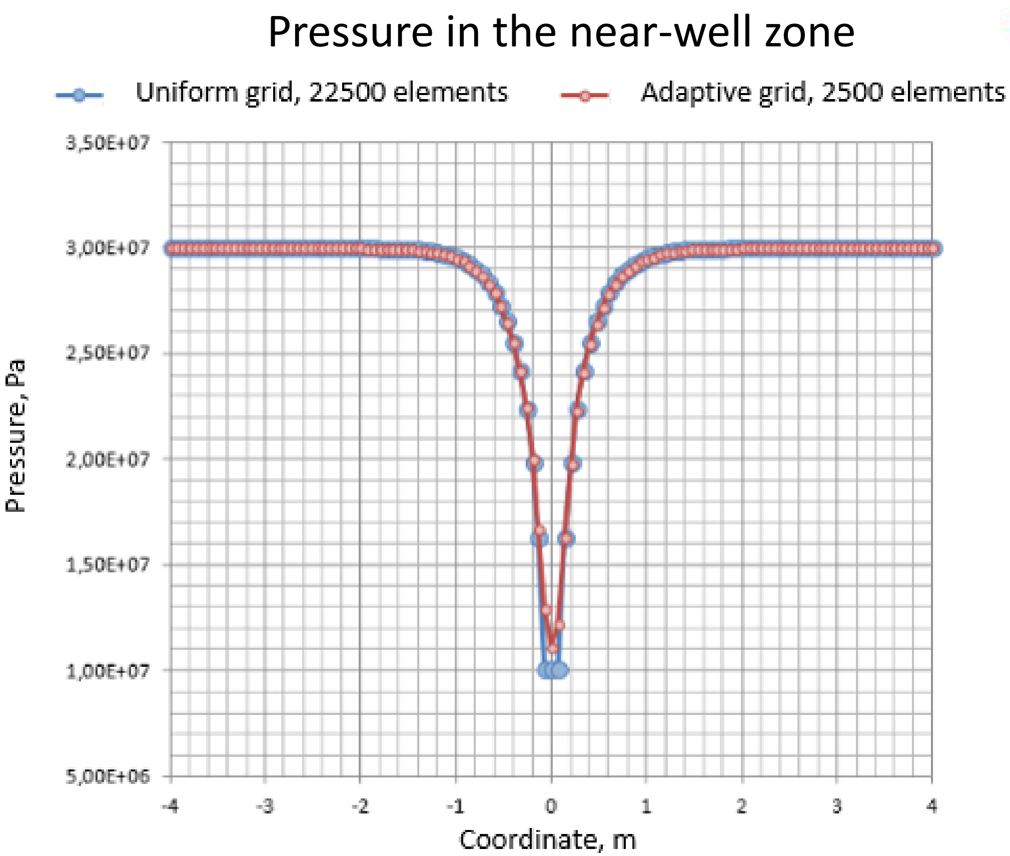
Weak formulation of the boundary value problem for finite elements method:

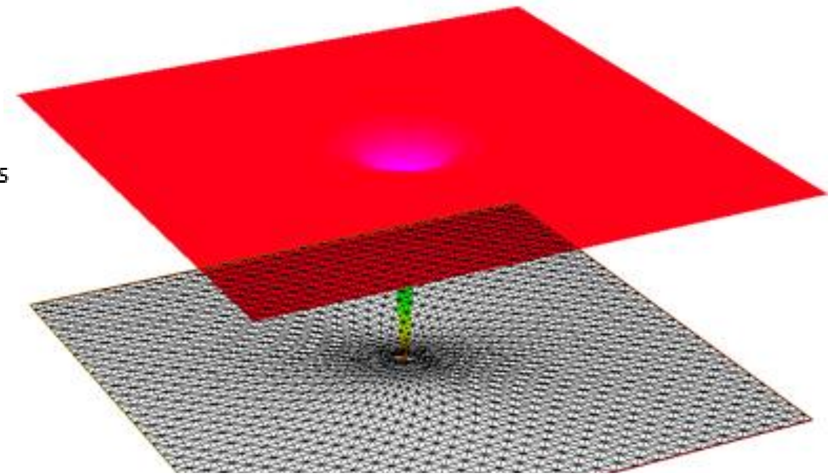
$$\sum_{i,j} \int_{\Omega_C} \frac{\partial \boldsymbol{x}}{\partial \xi_i} \cdot \frac{\partial}{\partial \xi_j} \left(A_{ij} \boldsymbol{v}\right) d\boldsymbol{\xi} + \sum_i \int_{\Omega_C} \frac{\partial \boldsymbol{x}}{\partial \xi_i} \left(B_i \boldsymbol{v}\right) d\boldsymbol{\xi} = 0.$$

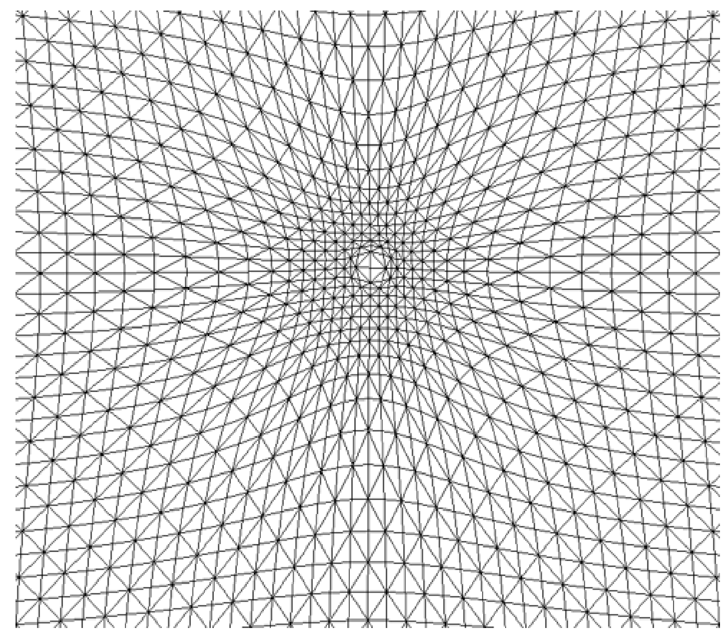
It is often sufficient the boundary conditions for the problem to set the immobility of the nodes on the boundary of the region.

*A detailed derivation of the equations is described in Weizhang Huang, Russell, Robert D.: Adaptive Moving Mesh Methods. Springer-Verlag, New York (2011)

Using Adaptive Grid For Geomechanical Problem







The calculations are performed with the use of the open-source FreeFem++ software package

Statement of Poroelasticity Problem

$$-G\nabla\cdot\left(\nabla u+(\nabla u)^T\right)-G\frac{2\nu}{(1-2\nu)}\nabla(\nabla\cdot u)+\alpha\nabla p=F \qquad \text{in } \Omega \times (0,T),$$

$$\frac{\partial}{\partial t}(S\mathrm{e}\,p+\alpha\nabla\cdot u)-\nabla\cdot\left(\frac{k}{\mu}\nabla p\right)=Q \qquad \text{in } \Omega \times (0,T),$$

$$u=0 \qquad \text{on } \Gamma_c,$$

$$\left[G\left(\nabla u+(\nabla u)^T\right)+G\frac{2\nu}{1-2\nu}\nabla\cdot uI\right]\hat{n}-\beta\alpha p\hat{n}\chi_{tf}=0 \qquad \text{on } \Gamma_t,$$

$$p=0 \qquad \text{on } \Gamma_d,$$

$$-\tfrac{\partial}{\partial t}\left((1-\beta)\alpha u\cdot\hat{n}\right)\chi+\tfrac{k}{\mu}\nabla p\cdot\hat{n}=h_1\chi_{tf} \qquad \text{on } \Gamma_f,$$

$$\mathrm{e}\,p+\alpha\nabla\cdot u=v_0 \qquad \text{in } \Omega \times \{0\},$$

$$(1-\beta)\alpha u\cdot\hat{n}=v_1 \qquad \text{on } \Gamma_{tf} \times \{0\}.$$

*The meaning of the remaining notation is presented in Chadia Affane Aji.: Poroelasticity. Auburn, Alabama (2007).

Weak Formulation of The Boundary Value Problem For Poroelasticity Using Adaptive Grids

$$\int_{\Omega}\left[G\left(\nabla u^{n+1}+\left(\nabla u^{n+1}\right)^{T}\right):\nabla v+G\frac{2\nu}{1-2\nu}\left(\nabla\cdot u^{n+1}\right)(\nabla\cdot v)\right]+\int_{\Omega}\alpha\nabla p^{n+1}v=$$

$$\int_{\Omega}F^{n+1}v+\int_{\Gamma_t}G\left(\nabla u^{n+1}+\left(\nabla u^{n+1}\right)^{T}\right)\cdot\hat{n}v+\int_{\Gamma_t}G\frac{2\nu}{1-2\nu}\left(\nabla\cdot u^{n+1}\right)\hat{n}\cdot v,$$

$$-\int_{\Omega}\alpha u^{n+1}\cdot\nabla q+\int_{\Omega}\left(Se\,p^{n+1}q+\frac{k\tau}{\mu}\theta\nabla p^{n+1}\cdot\nabla q\right)=$$

$$\int_{\Omega}\left(\tau\left(\theta Q^{n+1}+(1-\theta)Q^{n}\right)+\alpha\nabla\cdot u^{n}+Sep^{n}\right)q-\int_{\Omega}\frac{k\tau}{\mu}(1-\theta)\nabla p^{n}\nabla q$$

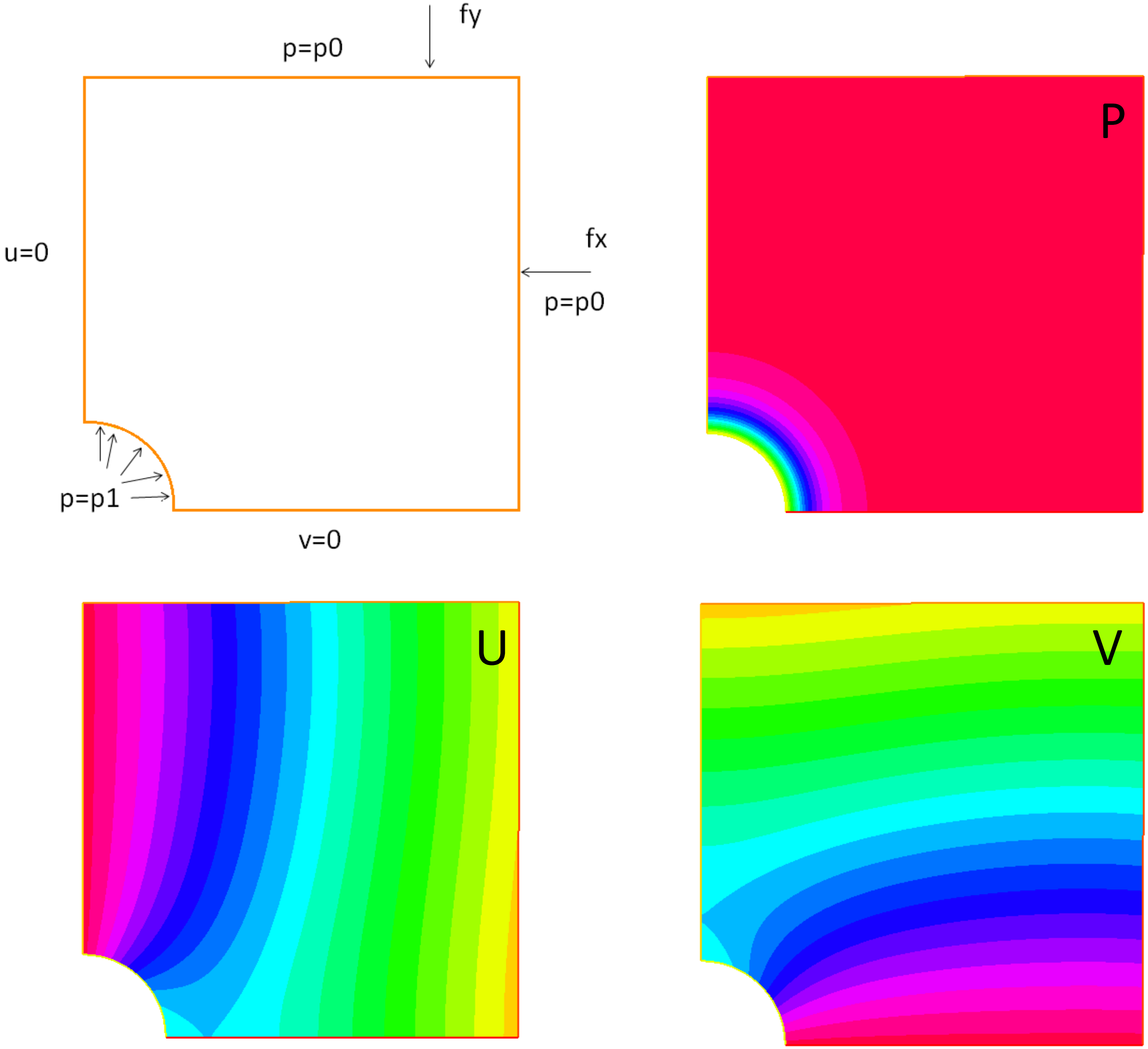
$$-\int_{\Gamma_f}\alpha u^{n+1}\cdot\hat{n}q+\int_{\Gamma_f}\frac{k}{\mu}\left(\theta\nabla p^{n+1}+(1-\theta)\nabla p^{n+1}\right)\cdot\hat{n}q$$

$$\sum_{i,j}\int_{\Omega_C}\frac{\partial\boldsymbol{x}^{\boldsymbol{n+1}}}{\partial\xi_i}\cdot\frac{\partial}{\partial\xi_j}\left(A_{ij}\boldsymbol{\omega}\right)d\boldsymbol{\xi}+\sum_{i}\int_{\Omega_C}\frac{\partial\boldsymbol{x}^{\boldsymbol{n+1}}}{\partial\xi_i}\left(B_i\boldsymbol{\omega}\right)d\boldsymbol{\xi}=0.$$

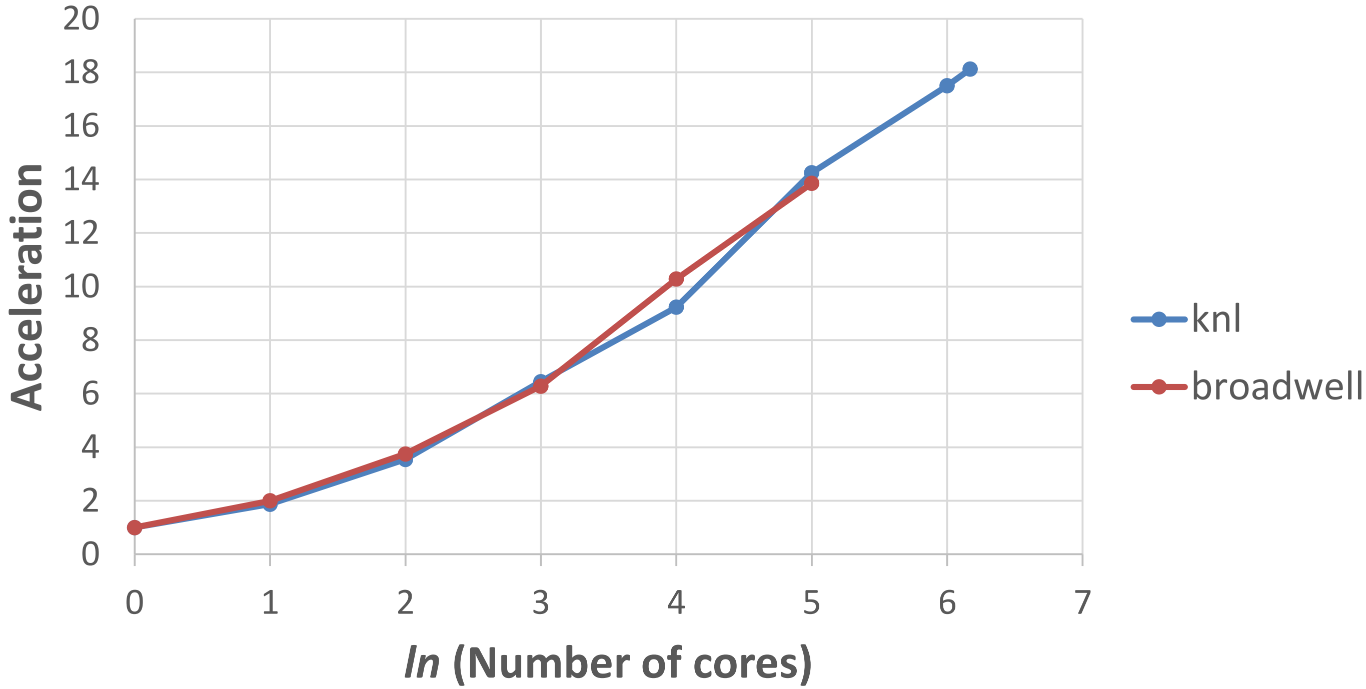
The Parallel Implementation of a Finite Element Method

- Decomposition of computational domain:
  - a regular grid – trivially
  - complex initial organization of the grid – Metis package
- The Schwarz algorithm with overlapping (two elements)
- Coarse grid preconditioner
- Approach of grid adaptation keeps the number of grid elements unchanged and does not require additional solving the problem of load balancing between MPI processes during computing

Test Computation For Poroelasticity



Scalability tests for poroelasticity



- Siberian Supercomputer Center (two 16-core Intel Broadwell / 72-core Intel KNL)
- A grid consisting of ~440000 triangles
- FreeFem ++ solver with an extended interface based on MPI

Testing the Intel Optane Memory Usage for Poroelasticity

Intel Optane

- a new SSD product
- based on the novel 3D XPointTM technology
- can be used instead of DRAM, but as a “slow” memory
- much cheaper than RAM per Gb

A node of SSCC equipped with Optane

Memory Configurations	Number of Triangles	Used Memory, Gb	Execution Time, s
DDR4	$12{,}5 \cdot 10^6$	51	941
DDR4	$32 \cdot 10^6$	125	5508
DDR4/Optane	$50 \cdot 10^6$	293	8373
DDR4/Optane	$72 \cdot 10^6$	466	19614

Conclusion

The use of adaptive grids for solving poroelasticity problems is discussed:

- An algorithm for constructing an adaptive grid with the Jacobian coordinate transformation is presented. The adaptation algorithm changes the grid density, which depends on the gradient of a desired function.
- The number of nodes of the adaptive mesh remains unchanged and is equal to the number of nodes of the initial grid.
- The results of the simulation by the finite elements method of the pressure distribution for the fluid filtration problem in the near-well zone using adaptive mesh are presented
- The studies of the parallel implementation for poroelasticity problems show the acceleration of about 14 times in 32 cores of Intel Broadwell and about 18 times in 72 cores of Intel KNL
- Intel Optan technology usage have been consideration

Thanks for your attention!